ECE317 : Feedback and Control

Lecture :
Relative stability

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Course roadmap

Modeling

- Laplace transform
- Transfer function
- Block Diagram
- Linearization
- Models for systems
  - electrical
  - mechanical
  - example system

Analysis

- Stability
  - Pole locations
  - Routh-Hurwitz
- Time response
  - Transient
  - Steady state (error)
- Frequency response
  - Bode plot

Design

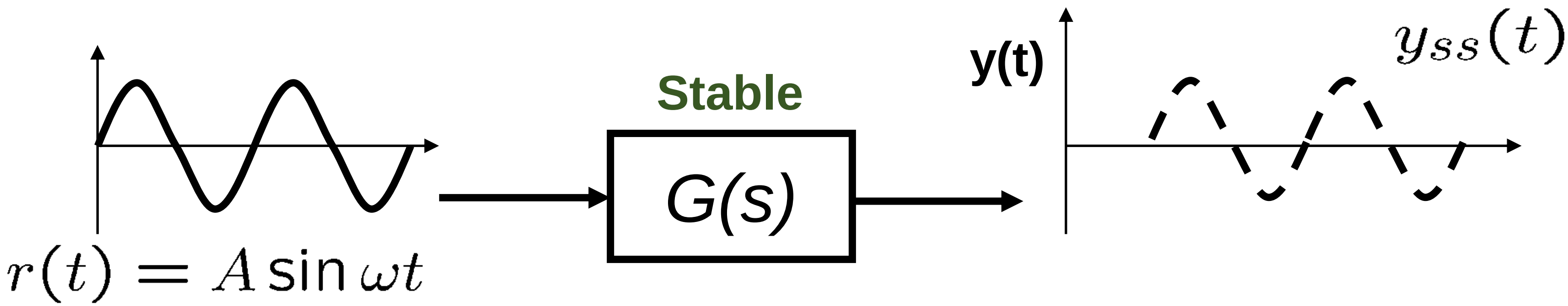
- Design specs
- Frequency domain
- Bode plot
- Compensation
- Design examples

Matlab & PECS simulations & laboratories

Frequency response (review)

- Steady state output $y_{ss}(t) = A|G(j\omega)|\sin(\omega t + \angle G(j\omega))$
  - Frequency is same as the input frequency ω
  - Amplitude is that of input (A) multiplied by $|G(j\omega)|$
  - Phase shifts $\angle G(j\omega)$

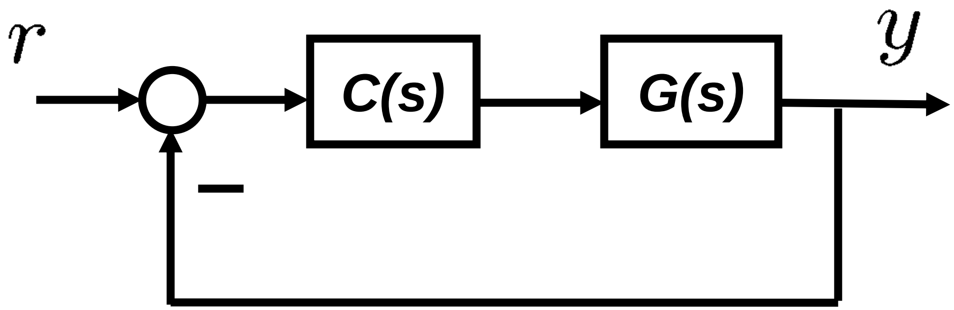
Gain



- Frequency response function (FRF): $G(j\omega)$
- Bode plot: Graphical representation of $G(j\omega)$

Stability of feedback system

- Consider the feedback system



- Fundamental questions
  - If *G* and *C* are stable, is the closed-loop system *always stable*?
  - If *G* and *C* are unstable, is the closed-loop system *always unstable*?

Closed-loop stability criterion

- Closed-loop stability can be determined by the roots of the characteristic equation

$$1 + L(s) = 0, \;\; L(s) := G(s)C(s)$$

- Closed-loop system is stable if the Ch. Eq. has all roots in the open left half plane.
- How to check the closed-loop stability?
  - Computation of all the roots
  - Routh-Hurwitz stability criterion
  - *Relative stability criterion (phase margin): Open-loop FRF $L(j\omega)$ contains information of closed-loop stability.*

Advantages of using frequency response to determine stability

- It does not require transfer functions, just experimental frequency response data of the (stable) open-loop system are necessary to judge the closed-loop stability. On the other hand, Routh-Hurwitz criterion needs transfer functions.
- It leads to the concept of "stability margin", i.e., gain-margin and phase-margin. From Routh-Hurwitz criterion, we can only judge "stable or not".

Remarks on stability margin criterion

- Stability margin criterion gives not only *absolute* but also *relative stability*.
  - Absolute stability: Is the closed-loop system stable or not? (Answer is yes or no.)
  - Relative stability: How "much" is the closed-loop system stable? (Margin of safety)
- Relative stability (stability margin) is important because a math model is never accurate.
- How to measure relative stability?
  - Gain margin (GM) & Phase margin (PM)

Gain margin (GM)

- Phase crossover frequency ω_p:

$$\angle L(j\omega_p) = -180^\circ$$

- Gain margin (in dB)

$$GM = 20\log_{10}\frac{1}{|L(j\omega_p)|}$$

  - Indicates how much OL gain can be multiplied without violating CL stability.

Phase margin (PM)

- Gain crossover frequency ω_g:

$$|L(j\omega_g)| = 1$$

- Phase margin

$$PM = \angle L(j\omega_g) + 180^\circ$$

  - Indicates how much OL phase lag can be added without violating CL stability.

Phase margin test for stability

- (Under a some conditions*) Closed loop stability of a system is guaranteed when

Phase margin is positive (PM > 0)

i.e. the phase of the system needs to be greater than -180 degrees at the gain crossover frequency

* i) there is exactly one gain crossover frequency

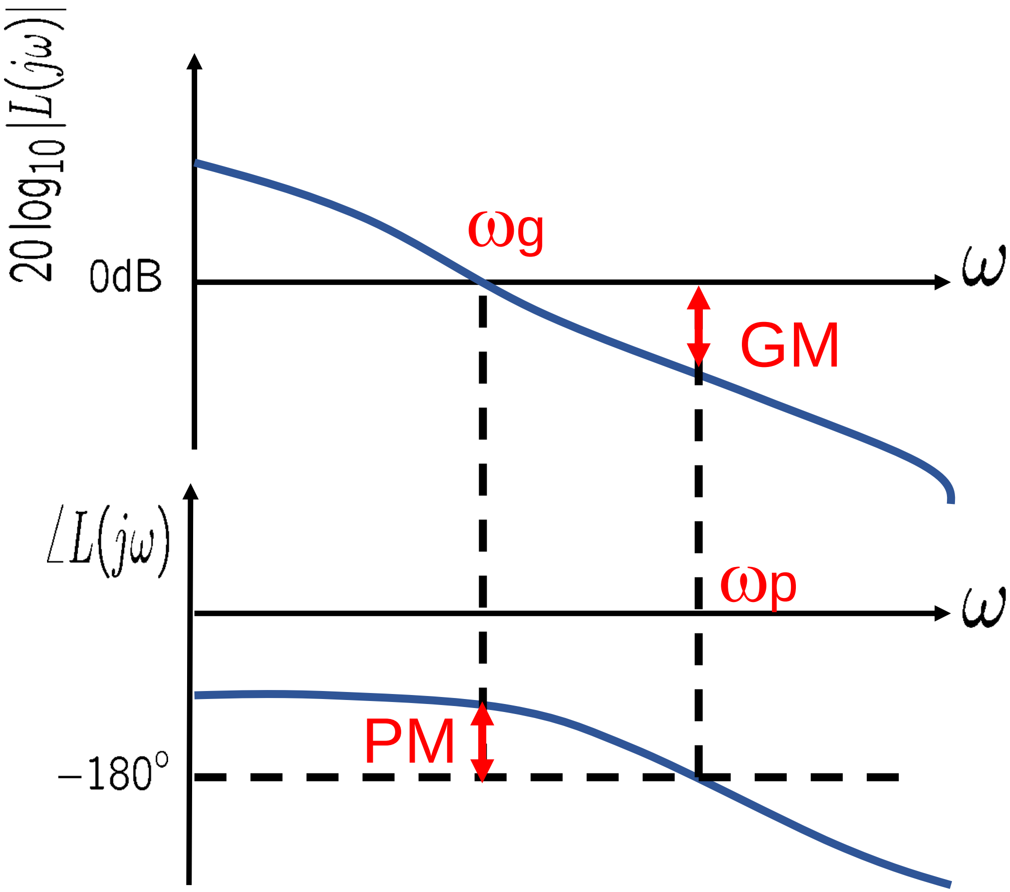
  ii) the system is open-loop stable

Phase margin test for stability

- Note under this test there is NO requirements for gain margin.
- However, it is generally stated that gain margin must also be positive. This can be shown to not be true by a counter example.

Relative stability on Bode plot

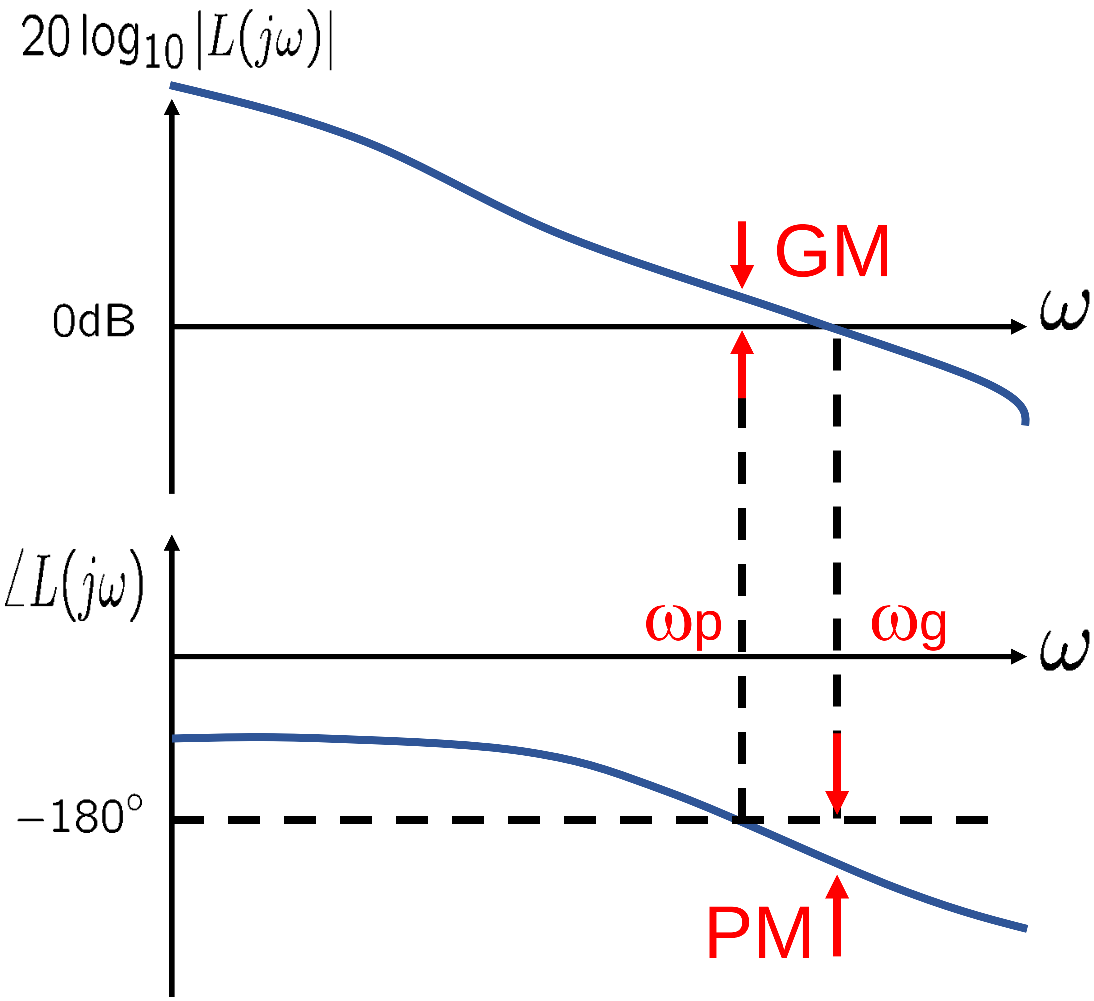
- When $\angle L(j\omega_g) > -180°$ → PM is positive otherwise, it is negative
- When $|L(j\omega_p)| < 0dB$ → GM is positive, otherwise it is negative



Here GM is positive

Here PM is positive

Unstable closed-loop case



Here GM is negative

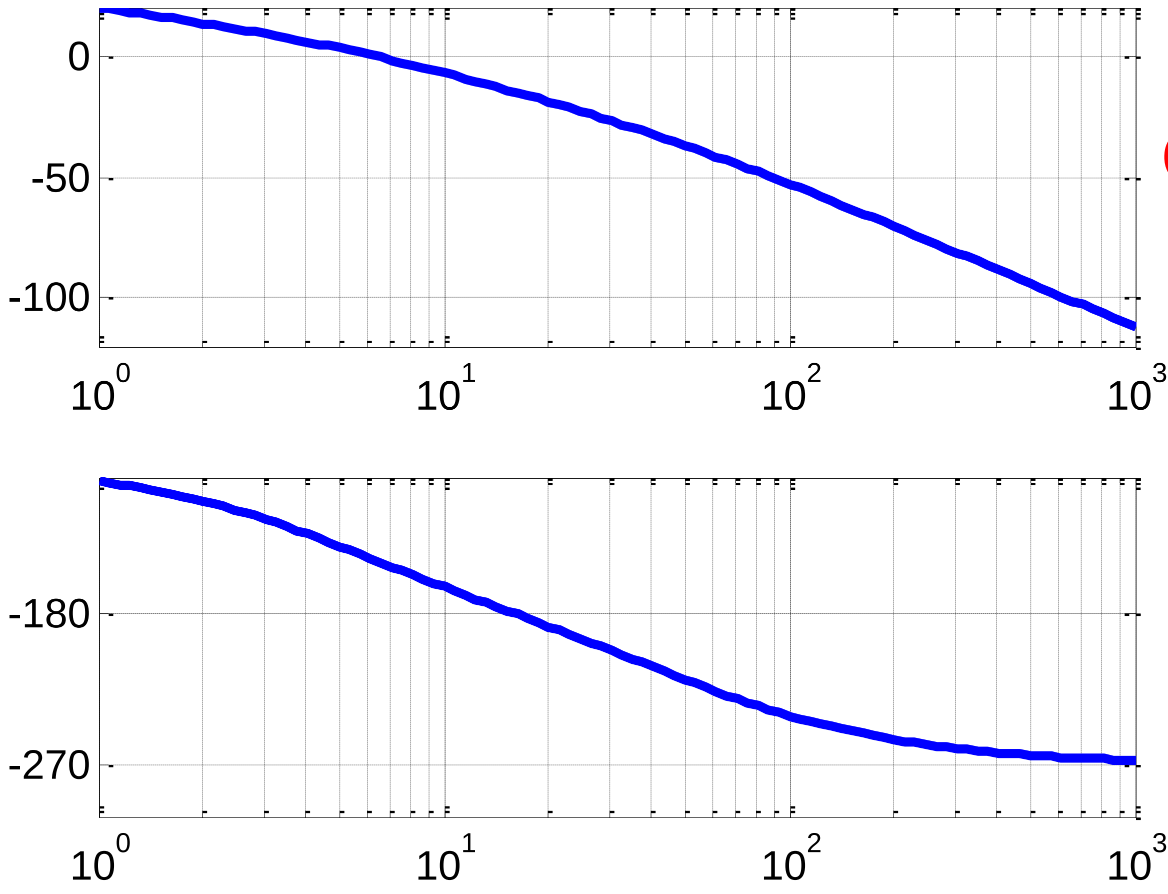
Here PM is negative

Notes on Bode plot

- Advantages
  - Without computer, Bode plot can be sketched easily by using straight-line approximations.
  - GM, PM, crossover frequencies are easily determined on Bode plot.
  - Controller design on Bode plot is simple.
- Disadvantage
  - If OL system has poles in open right half plane, it will be complicated to use Bode plot for closed-loop stability analysis.

Example 1:

$$L(s) = \frac{2500}{s(s+5)(s+50)}$$



ωg = ?

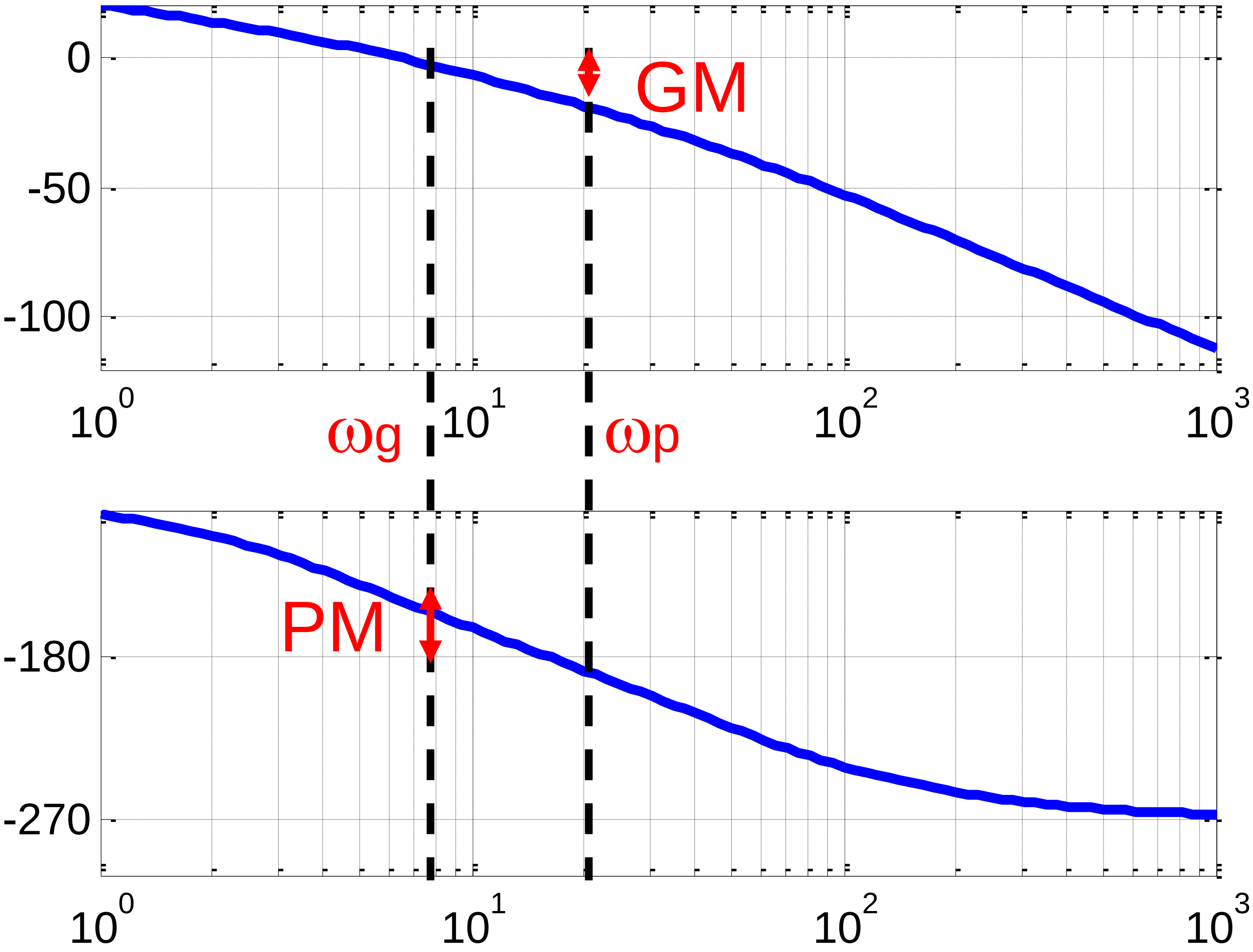
GM: positive? negative?

ωp= ?

PM: positive? negative?

Example, cont'd:

$$L(s) = \frac{2500}{s(s+5)(s+50)}$$



ωp = 16 rds/s

GM: positive

ωg= 6 rds/s

PM: positive

=> closed-loop system is STABLE

Example, cont'd: $L(s) = \frac{2500}{s(s+5)(s+50)}$

- Using straight line asymptotic approximation determine:

i. Unity gain crossover frequency: ωg

ii. Phase margin: PM

iii. -180 degree phase crossover frequency: ωp

iv. Gain margin: GM

- confirm the results with Matlab 'margin' command

(This example is worked out in class and homework)

Example 2:

Sketch the asymptotic Bode plot for the following loop gain.
Annotate the plots completely:

1) Show the values of all break frequencies for magnitude and phase,
2) For magnitude plots: show i) gain along all straight line segments, and ii) slopes,
3) For phase plots: show the slopes.

$$L(s) = \frac{A}{\left(1 + \frac{s}{\omega_0}\right)\left(1 + \frac{s}{\omega_1}\right)}$$

where:

$$A = 200, \quad \omega_0 = 100, \quad \omega_1 = 300$$

(This example is worked out in class and homework)

Example 3:

Sketch the asymptotic Bode plot for the following loop gain.
Annotate the plots completely and sketch using frequency in Hz (not rds/s):

1) Show the values of all break frequencies for magnitude and phase,
2) For magnitude plots: show i) gain along all straight line segments, and ii) slopes,
3) For phase plots: show the slopes.

$$L(s) = \frac{A\left(1 - \frac{s}{\omega_z}\right)}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

where:

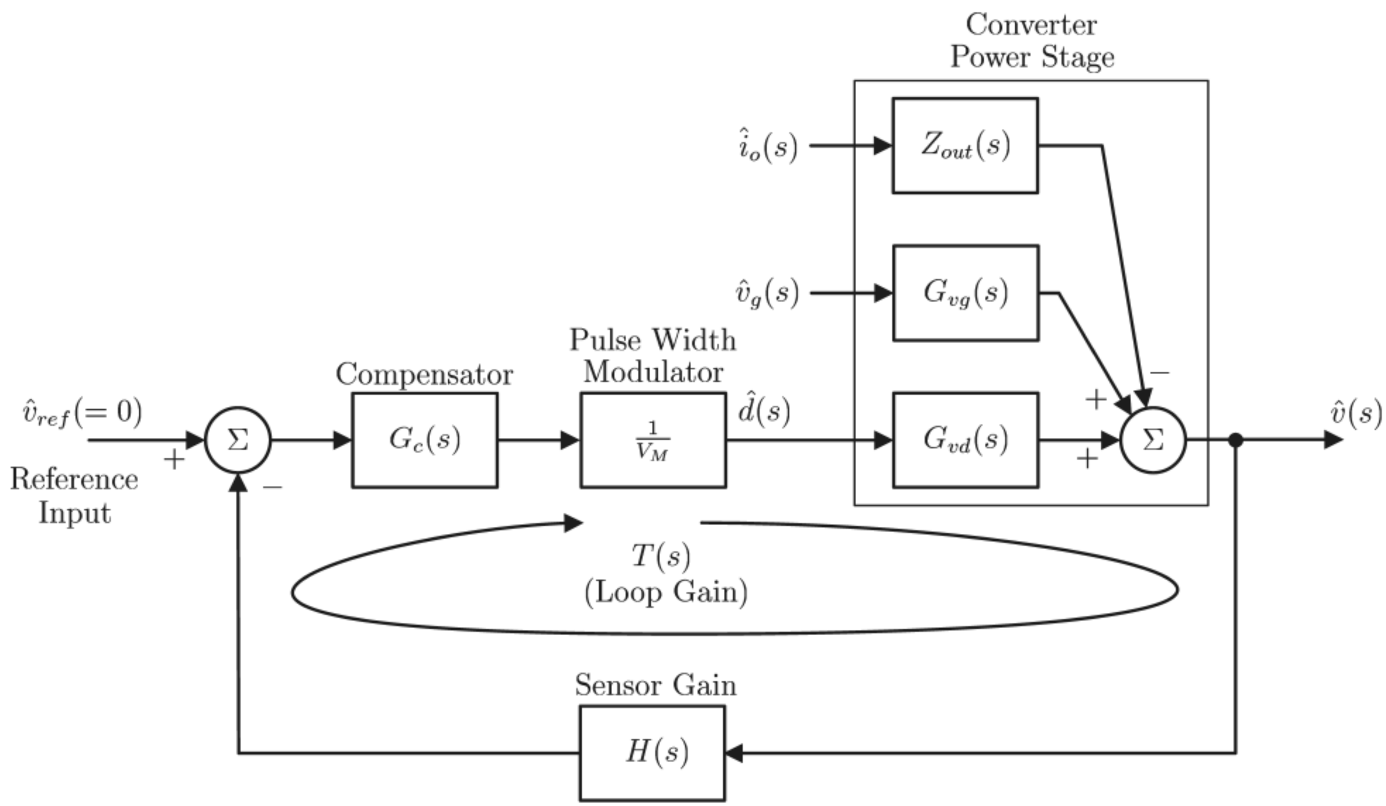
$$A = 120, \quad \omega_z = 2\pi(2500), \quad \omega_0 = 2\pi(500), \quad Q = 5$$

(This example is worked out in class and homework)

Summary

- Relative stability:
  - Gain margin, phase crossover frequency
  - Phase margin, gain crossover frequency
- Relative stability on Bode plot
- We normally emphasize PM in controller design.

Application to the lab:



$$T(s) = \frac{1}{V_M} \cdot G_c(s) \cdot G_{vd}(s) \cdot H(s)$$

In the lab *T*(*s*) is used to refer to the loop gain *L*(*s*)

Application to the lab. Cont'd

Block diagram reduction leads to the closed loop transfer functions:

$$\hat{v} = G_{vref\_CL}(s)\hat{v}_{ref} + G_{vg\_CL}(s)\hat{v}_g - Z_{out\_CL}(s)\hat{i}_o$$

$$G_{vref\_CL}(s) = \frac{1}{H(s)}\frac{T(s)}{1+T(s)}$$

$$G_{vg\_CL}(s) = \frac{G_{vg}(s)}{1+T(s)}$$

$$Z_{out\_CL}(s) = \frac{Z_{out}(s)}{1+T(s)}$$

where:

$$T(s) = \frac{1}{V_M} \cdot G_c(s) \cdot G_{vd}(s) \cdot H(s)$$

Application to the lab. Cont'd

$$T(s) = \frac{1}{V_M} \cdot G_c(s) \cdot G_{vd}(s) \cdot H(s)$$

- To determine absolute stability of this system we can use Routh-Hurwitz criterion. Note however, this is NOT applied to $T(s)$, but rather the Routh-Hurwitz criterion is applied to the denominator polynomial of

$$\frac{1}{1+T(s)} \quad \text{or} \quad \frac{T(s)}{1+T(s)}$$

- To determine absolute stability and relative stability of the system we find the phase and gain margins exhibited by the loop gain $T(s)$